On radiative widths and a pattern of quark-diquark-gluon configuration mixing in low-lying scalar mesons

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The earlier suggested and developed idea of quark-hadron duality, underlying "bremsstrahlung-weighted" sum rules for total or polarized photon interaction cross sections, is applied to the description of excitation of light scalar mesons in gamma-gamma interactions. The emphasis is put on the discussion of a role of the scalar diquark cluster degrees of freedom in the radiative formation of light scalar mesons.

1. Introduction

The (constituent) quark hadron duality sum rules, used here, follow from the assumed equivalence of two complete sets of state vectors, saturating certain integral sum rules, one of the sets being the solution of the bound state problem with colour-confining interaction, while the other describes free partons. The sum rules satisfying the assumed duality condition have been chosen to be those related to fluctuation of the relativistic electric dipole moment (EDM) operator in the configuration space of valence partons in a given system taken in the "infinite momentum" frame. The relevance of these sum rules has been tested in some models of quantum field theory [1] and used to derive a number of seemingly successful relations for the hadron electromagnetic radii [2] and two-photon decay widths of the lowest spin meson resonances [3]. The two-photon-meson couplings give rather direct information on the flavour content of considered states. Moreover, the salient feature of our sum rule approach enables one to put forward most distinctly the flavour content of the states at hand and, at the same time, somehow to circumvent many of model-dependent aspects of the detailed structure and dynamics of multi-component bound quark-gluon states. Therefore they could be especially useful in the case of hadrons with the complex and poorly understood constituent structure. In what follows, we only mention briefly some technical details. Varying the polarizations of colliding photons, one can show that a linear combination of certain $\gamma\gamma \to q\bar{q}$ cross-sections will dominantly collect the $q\bar{q}$- states with definite spin-parity and hence the low-mass meson resonances with the same quantum numbers. The polarization structure of the transition matrix element $M(J^{PC} \leftrightarrow 2\gamma)$

for the scalar meson resonance

$$M(0^{++} \leftrightarrow 2\gamma) \;=\; G[(\epsilon_1\epsilon_2)(k_1k_2) - (\epsilon_1k_2)(\epsilon_2k_1)] \qquad (1)$$

where k_i^μ are the momenta of photons, ϵ_i^ν are polarization vectors of photons, G is a constant proportional to the coherent sum of amplitudes describing the two-photon annihilation of partons, composing a given meson. By definition

$$|G| = 64\pi\Gamma_{\gamma\gamma}/m^3, \qquad (2)$$

where m is the meson mass and $\Gamma_{\gamma\gamma}$ is the two-photon width of a given meson. Introducing the $\gamma\gamma$ - cross-sections $\sigma_{\perp(\parallel)}$ (and the integrals thereof) that refer to colliding plane-polarized photons with the perpendicular (parallel) polarizations, and σ_p corresponding to circularly polarized photons with parallel spins, one can then show that the combinations of the integrals over the bremsstrahlung-weighted and polarized $\gamma\gamma \to q\bar{q}$ cross-sections, $I_\perp - (1/2)I_p$, $I_\parallel - (1/2)I_p$, I_p will be related to low-mass meson resonances having spatial quantum numbers $J^{PC} = 0^{-+}$ and 2^{-+}, 0^{++} and $2^{++}(\lambda = 0)$, $2^{++}(\lambda = 2)$, if we confine ourselves to the mesons with spins $J \leq 2$ ($\lambda = 0$ or 2 being the z-projection of the total angular momentum of the tensor mesons). In what follows, we focus mainly on scalar meson sum rules in the light quark sector. As is known, the long-lasting experimental efforts have presently resulted in identification of a few scalar states with masses below $2GeV$, labelled by isospin [4,5]:

$$I=0 : f_0(600)\ or\ \sigma(500), f_0(980), f_0(1200 \div 1500), f_0(1506), f_0(1710);$$
$$I=1/2 : \kappa(800), K_0^*(1430);$$
$$I=1 : a_0(980), a_0(1450).$$

Following [6,7] we assume that the listed resonances are interpreted as two meson nonets and a scalar glueball with the mixed valence quark and gluon configurations. In the states lying above 1 GeV the dominant configuration is, presumably, a conventional $q\bar{q}$ nonet mixed with the glueball of (quenched) lattice QCD. Below 1 GeV the states also form a nonet, where the central binding role is played by the $SU(3)_{c(f)}$ -triplet diquark clusters $(qq)_{\bar{3}}(\bar{q}\bar{q})_3$ in S-wave with some $q\bar{q}$ admixtures in P-wave, and maybe less important glueball part in their state vectors. We begin with consideration of only constituent quark and scalar diquark as the basic degrees of freedom in the first stage of $\gamma\gamma$ - reactions: $\gamma + \gamma \to q + \bar{q}, (qq) + (\bar{q}\bar{q})$ where the quark and diquark are treated as elementary structureless spinor and scalar massive particles with "minimal" electromagnetic interaction. Evaluating cross-sections and elementary integrals we get the sum rules for radiative widths of resonances with $J^{PC} = 0^{++}$

$$\sum_i \frac{\Gamma(S_i \to 2\gamma)}{m_{S_i}^3} \simeq \sum_q I_S(q) + \sum_{qq} I_S(qq), \qquad (3)$$

where

$$I_S(q) = \frac{3}{16\pi^2}\langle Q(q)^2\rangle^2 \frac{5\pi\alpha^2}{9m_q^2}, \qquad (4)$$

$$I_S(qq) = \frac{3}{16\pi^2}\langle Q(qq)^2\rangle^2 \frac{2\pi\alpha^2}{9m_{qq}^2}. \qquad (5)$$

All the integrals over the parton (that is the quark and diquark) production cross

sections are rapidly converging and all the resonance cross sections are taken in the narrow width approximation so that for the wide scalar mesons the masses in Eq.(3) have rather the meaning of the "mean value"-masses.

The term $I_S(qq)$ in Eqs.(3) and (5) corresponds to ascribing a possible role to scalar diquarks as a constituent triplet $(\bar{d}\bar{s}), (\bar{u}\bar{s}), (\bar{u}\bar{d})$ of "partons" with respective masses and electric charges composing, at least in part, the scalar meson nonets.

Assuming now for $a_0(980)$ either of two limiting options: (a)- the isovector quark-antiquark $\bar{q}q$-structure, or (b)- the isovector diquark-antidiquark $(\bar{q}\bar{s})(qs)$ -configuration, and using Eq.(3)-(5), one gets

$$\Gamma_{\gamma\gamma}(a_0(980)) \simeq 1.6\ (.12)\ keV \tag{6}$$

The lower value of the width in (6) is obtained if, following [8], we accept for the diquark masses $m_{qs} \simeq 560$ MeV and $m_{ud} \simeq 320$ MeV (those seem to be of minimal value as compared to fitted mass values of many other models, thus stressing maximally the role of the diquark configurations in various hadrons), while the masses of light quarks are taken to be $m_{u,d} \simeq 240$ MeV and $m_s \simeq 350$ MeV, according to [2].

Both values in Eq.(6) are different from $\Gamma_{\gamma\gamma}(a_0(980)) = .30 \pm .10$ keV [9], that is in between two. Another evidence against the interpretation of $a_0(980)$ -meson as a usual $\bar{q}q$-state is the quenched LQCD evaluation of the scalar, isovector quarkonium mass $m(0^{++}, I^G = 1^-) = 1.330(50)\ GeV$ [10]. Therefore, it is quite natural to suppose the $a_0(980)$- meson to have a mixed structure (together with its higher-lying partner)

$$|a_0(1474)\rangle = cos\theta|(1/\sqrt{2})(u\bar{u} - d\bar{d})\rangle + \\ +sin\theta|(1/\sqrt{2})((\bar{d}\bar{s})(ds) - (\bar{u}\bar{s})(us))\rangle, \tag{7}$$

$$|a_0(985)\rangle = -sin\theta|(1/\sqrt{2})(u\bar{u} - d\bar{d})\rangle + \\ +cos\theta|(1/\sqrt{2})((\bar{d}\bar{s})(ds) - (\bar{u}\bar{s})(us))\rangle. \tag{8}$$

By convention, we take here the phases of $\sqrt{I_S(q)}$ and $\sqrt{I_S(qq)}$ as $+1$ and-1, respectively. Further, with the fit $\theta \simeq 10^o$ to reproduce $\Gamma_{\gamma\gamma}(a_0(980)) \simeq .3$ keV, one gets also $\Gamma_{\gamma\gamma}(a_0(1474)) \simeq 4.6$ keV, which should be tested experimentally yet.

2. A model of mixing matrices for light scalar mesons

We turn now to a "reconstruction" of the bare masses of two (finally) mixed scalar nonets. Taking for granted the physical masses 1474 MeV - for higher-mass, $q\bar{q}$-dominant isovector meson and 985 MeV - for lower-mass $(qq)(\bar{q}\bar{q})$ one, and having defined $\theta \simeq 10^o$, the diagonal and nondiagonal elements in the 2×2 mass-matrix of the isovector states are easily derived to be $M(I = 1; (q\bar{q})) = 1434$ MeV, $M(I = 1; ((qs)(\bar{q}\bar{s}))) = 1005$ MeV, and the universal, with the tentatively assumed $SU(3)$-symmetry, nondiagonal "mass" $h = 95$ MeV. In fact, it represents the transition coupling between states of two multiplets.

The "bare" masses of the isospinor states are $M(I = 1/2; (q\bar{s})) \simeq 1435$ MeV and $M(I = 1/2; ((ud)(\bar{q}\bar{s}))) \simeq 812$ MeV. They correspond to "physical" masses

$m \simeq 1450$ MeV and $m \simeq 790$ MeV, according to latest data [4,5].

At last, to define the mass of the lightest, isoscalar "bare" state we invoke the mass formula of the ideal-mixing-form

$$M((ud)(\bar{u}\bar{d})) = 2M((ud)(\bar{u}\bar{s})) - \\ -M((qs)(\bar{q}\bar{s})) \simeq 620\ MeV. \quad (9)$$

The mixing of a glueball and 2 pairs of isoscalar mesons is described by the following mass matrix, which is diagonalized by the masses of 5 physical states:

$$\begin{pmatrix} M_G & f & f\sqrt{2} & g & g\sqrt{2} \\ f & M_{S_1} & 0 & h\sqrt{2} & 0 \\ f\sqrt{2} & 0 & M_{N_1} & h & h\sqrt{2} \\ g & h\sqrt{2} & h & M_{S_2} & 0 \\ g\sqrt{2} & 0 & h\sqrt{2} & 0 & M_{N_2} \end{pmatrix} \quad (10)$$

$$\Longrightarrow \text{diag}\ (m_1, m_2, m_3 m_4, m_5).$$

M_G and M_{S_1,N_1} (or M_{S_2,N_2}) stand for the mass of the primitive glueball, and $S_1 = s\bar{s}$ and $N_1 = n\bar{n} \equiv (u\bar{u} + d\bar{d})/\sqrt{2}$ (or $S_2 = ((\bar{n}\bar{s})(ns) \equiv ((\bar{d}\bar{s})(ds) + (\bar{u}\bar{s})(us))/\sqrt{2}$ and $N_2 = (\bar{u}\bar{d})(ud)))$ mesons, respectively, the subscripts 1 or 2 indicating the quark (or diquark) composition of the nonet the state belongs to; m_i stand for the masses of the physical states; f (or g) is the glueball–$q\bar{q}(or(\bar{q}\bar{q})(qq))$–meson coupling and h is the nondiagonal quark-to-diquark pair transition coupling. Following [11], we take all couplings having dimensionality $(mass)$, in accord with the dimensionality of the diagonal entries of (10). All quantities in (10) are considered to be real numbers.

The mixing between the glueball and the low-lying $(\bar{q}\bar{q})(qq)$-states can be less important also due to relative smallness of the lowest order gg-to-$(\overline{qq})(qq)$ transition amplitude as compared to the gg-to-$\bar{q}q$ transition. The relevance of these arguments is illustrated also by the (approximate) validity of mass-formulae, Eq.(9), which could be strongly violated if the annihilation-induced mixing of different flavours would take place. Therefore, we neglect, as a first approximation, the coupling g in the general 5×5 mass-matrix. Defining the relation between the physical and bare states

$$\begin{pmatrix} \underline{f_0(1710)} \\ \underline{f_0(1506)} \\ f_0(m_3) \\ \underline{f_0(980)} \\ f_0(m_\sigma) \end{pmatrix} = U(5) \begin{pmatrix} G \\ S_1 \\ \underline{N_1} \\ \underline{S_2} \\ \underline{N_2}, \end{pmatrix}, \quad (11)$$

where masses of the underlined states are considered to be defined and

$$U(5) = \begin{pmatrix} x_1 & y_1 & z_1 & u_1 & v_1 \\ x_2 & y_2 & z_2 & u_2 & v_2 \\ x_3 & y_3 & z_3 & u_3 & v_3 \\ x_4 & y_4 & z_4 & u_4 & v_4 \\ x_5 & y_5 & z_5 & u_5 & v_5 \end{pmatrix}, \quad (12)$$

we obtain the expression for the individual two-photon width of a scalar meson in the form

$$\Gamma_{\gamma\gamma}(f_0(m_i)) = m_i^3(125\alpha^2/7776\pi) \times \\ \times(y_i A_{S_1} + z_i A_{N_1} + u_i A_{S_2} + \\ +v_i A_{N_2})^2, \quad (13)$$

$$A_{S_1} = \sqrt{2}/(5m_s), A_{N_1} = 1/m_q, \quad (14)$$

$$A_{S_2} = -\sqrt{2}/(\sqrt{5}m_{qs}), \\ A_{N_2} = -2/(5\sqrt{5}m_{ud}), \quad (15)$$

the coefficients $y_i, .., v_i$ being the probability amplitudes to find the quark con-

figurations S_1, N_1, S_2, N_2 in the state vector of the (iso)scalar meson $f_0(m_i)$ with mass m_i. The minus signs in front of A_{N_2,S_2} in Eq.(15) is the reflection of our convention about opposite signs of the square-roots $\sqrt{I_{S,N}(q)}$ and $\sqrt{I_{S,N}(qq)}$, defined in Eq.(5) and effectively representing the fermion-quark and boson-diquark loops in the meson-two-photon transition diagrams. The orthogonality of the matrix U in Eq.(12) provides the "inclusive" sum rule to be fulfilled

$$\sum_i \frac{\Gamma_{\gamma\gamma}(f_0(m_i))}{m_i^3} = |A_{S_1}|^2 + |A_{N_1}|^2 + \\ + |A_{S_2}|^2 + |A_{N_2}|^2 \quad (16)$$

With assumed $g \simeq 0$, the unknown elements in the mass-mixing-matrix (10) are the coupling f and masses M_G and M_{S_1}, while among the physical masses the essentially unknown is a mass $1.2 \leq m(3) \leq 1.5$ GeV [4]. It seems worthwhile to mention that starting with the evident constraint $f^2 \geq 0$, we have obtained the reasonable bounds for these three quantities (in units of GeV)

$$1.31 \leq m(3) \leq 1.55, 1.47 \leq M_G \leq 1.51, \\ 1.49 \leq M_{S_1} \leq 1.69, (17)$$

just from basic secular equations with using the known masses of the physical mesons. The lower-bound of M_G and upper-bound of M_{S_1} in (17) are close to the values of respective masses found in [11] while their $m(3) = 1.26$ GeV is somewhat beyond of our more general bound.

3. Concluding remarks

The main results of this work are the following. We applied the idea of the $(q\bar{q}) - (\bar{q}\bar{q})(qq)$-configuration-mixing to a simpler isovector sector of two low-lying scalar nonets to fit the two-photon width of $a_0(980)$ and extract thereby the nondiagonal element in the mass-mixing-matrix. From the resulting more general (iso)scalar-mass 5×5-matrix, we derived the bounds on missing masses M_G and M_{S_1} of the "bare" glueball and scalar strangeonium states and poor defined mass of the so-called $f_0(1370)$-resonance. Under the assumptions that the bare glueball-two-diquark and also higher-lying "strangeonium"$(s\bar{s})$-two-diquark mixing can be neglected we obtained [12] masses of lowest $f_0(590)$,$f_(986)$ and $f_0(1470)$ (iso)scalar resonances in reasonable agreement with latest data of the E-791 and FOCUS Collaborations [5]. The preliminary estimates of the two-photon decay widths can be confronted only with the experimental value $.56 \pm .11$ keV for $f_0(986)$-meson and,with stated reservations [4], for the $f_0(m_3)$-resonance, where it is in the range of $3.8 \pm 1.5 \div 5.4 \pm 2.3$ keV. The theoretical estimates via dual sum rules [12] are, roughly, two times larger than the cited data. Clearly, for more quantitative statements we have to have new and more accurate $\gamma\gamma$ data.

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